NAG Fortran Library Routine Document

G05GAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

G05GAF generates a random orthogonal matrix.

2 Specification

```
SUBROUTINE G05GAF(SIDE, INIT, M, N, A, LDA, WK, IFAIL)
INTEGER            M, N, LDA, IFAIL
real               A(LDA,N), WK(*)
CHARACTER*1        SIDE, INIT
```

3 Description

G05GAF pre- or post-multiplies an m by n matrix A by a random orthogonal matrix U, overwriting A. The matrix A may optionally be initialised to the identity matrix before multiplying by U, hence U is returned. U is generated using the method of Stewart (1980). The algorithm can be summarized as follows.

Let $x_1, x_2, \ldots, x_{n-1}$ follow independent multinormal distributions with zero mean and variance $I\sigma^2$ and dimensions $n, n-1, \ldots, 2$; let $H_j = \mathrm{diag}(I_{j-1}, H_j^*)$, where I_{j-1} is the identity matrix and H_j^* is the Householder transformation that reduces x_j to $r_{jj}e_1$, e_1 being the vector with first element one and the remaining elements zero and r_{jj} being a scalar, and let $D = \mathrm{diag}(\mathrm{sign}(r_{11}), \mathrm{sign}(r_{22}), \ldots, \mathrm{sign}(r_{nn}))$. Then the product $U = DH_1H_2 \ldots H_{n-1}$ is a random orthogonal matrix distributed according to the Haar measure over the set of orthogonal matrices of n. See Theorem 3.3 in Stewart (1980).

4 References

Stewart G W (1980) The efficient generation of random orthogonal matrices with an application to condition estimates *SIAM J. Numer. Anal.* **17** 403–409

5 Parameters

1: SIDE – CHARACTER*1 *Input*

On entry: indicates whether the matrix A is multiplied on the left or right by the random orthogonal matrix U.

If SIDE = 'L', the matrix A is multiplied on the left, i.e., pre-multiplied.

If SIDE = 'R', the matrix A is multiplied on the right, i.e., post-multiplied.

Constraint: SIDE = 'L' or 'R'.

2: INIT – CHARACTER*1 *Input*

On entry: indicates whether or not A should be initialised to the identity matrix.

If INIT = 'I', then A is initialised to the identity matrix.

If INIT = 'N', then A is not initialised and the matrix A must be supplied in A.

Constraint: INIT = 'I' or 'N'.

3: M – INTEGER *Input*

On entry: the number of rows of the matrix A, m.

Constraint: if SIDE = 'L', then M > 1; otherwise M ≥ 1.

4: N – INTEGER *Input*

On entry: the number of columns of the matrix A, n.

Constraint: if SIDE = 'R', then N > 1; otherwise N ≥ 1.

5: A(LDA,N) – ***real*** array *Input/Output*

On entry: if INIT = 'N' then A must contain the matrix A.

On exit: the matrix UA when SIDE = 'L' or the matrix AU when SIDE = 'R'.

6: LDA – INTEGER *Input*

On entry: the first dimension of the array A as declared in the (sub)program from which G05GAF is called.

Constraint: LDA ≥ M.

7: WK(*) – ***real*** array *Workspace*

Note: the dimension of the array WK must be at least 2 × M if SIDE = 'L' or 2 × N if SIDE = 'R' .

8: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, −1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL = 0 unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value −1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value −1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL = 0 or −1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL = 1

On entry, M < 1,
or N < 1,
or LDA < M.

IFAIL = 2

On entry, SIDE ≠ 'L' or 'R',
or INIT ≠ 'I' or 'N'.

IFAIL = 3

On entry, an orthogonal matrix of dimension 1 has been requested.

7 Accuracy

The maximum error in $U^T U$ should be a modest multiple of ***machine precision***.

8 Further Comments

G05GBF computes a random correlation matrix from a random orthogonal matrix.

9 Example

A 4 by 4 orthogonal matrix is generated using the INIT = 'I' option and the result printed.

The generator mechanism used is selected by an initial call to G05ZAF.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     G05GAF Example Program Text
*     Mark 20 Revised. NAG Copyright 2001.
*     .. Parameters ..
      INTEGER          N, M, LDA
      PARAMETER        (N=4,M=4,LDA=10)
      INTEGER          NOUT
      PARAMETER        (NOUT=6)
*     .. Local Scalars ..
      INTEGER          I, IFAIL, J
*     .. Local Arrays ..
      real             A(LDA,N), WK(2*N)
*     .. External Subroutines ..
      EXTERNAL         G05CBF, G05GAF, G05ZAF
*     .. Executable Statements ..
      CALL G05ZAF('O')
      WRITE (NOUT,*) 'G05GAF Example Program Results'
      WRITE (NOUT,*)
*
      CALL G05CBF(0)
*
      IFAIL = 0
*
      CALL G05GAF('Right','Initialize',M,N,A,LDA,WK,IFAIL)
*
      DO 20 I = 1, M
         WRITE (NOUT,99999) (A(I,J),J=1,N)
   20 CONTINUE
      STOP
*
99999 FORMAT (1X,4F9.3)
      END
```

9.2 Program Data

None.

9.3 Program Results

```
 G05GAF Example Program Results

    -0.461    0.823   -0.251    0.218
     0.446    0.470    0.064   -0.759
    -0.766   -0.204    0.256   -0.554
     0.056    0.245    0.931    0.264
```